time in the wind tunnel. These data could be correlated with the extensive acoustic and hot-wire fluctuation studies currently in progress in transonic facilities. The precision curve given in Fig. 1 would apply for this application with an additional constraint on the fluctuation frequency that could be detected. Fundamental flow response phenomena constrain the frequency domain over which this technique could be applied. To assess the flow response limitations, some insight gained from the tests in the aeroballistic range can be utilized.

Drag forces decelerate the body during an aeroballistic range flight and the rate of deceleration is dependent on the body mass (inertia) as well as the drag force itself. At Mach numbers below 1.01, the shock detachment distance becomes large and does not respond instantaneously to the Mach number decrease of the body if the rate of deceleration is too great. Referring to Fig. 2, the response time can be expressed, in a simplified form, as

$$t_r = \Delta/(a-u_\ell) + \Delta/(a+u_\ell)$$

where a is the speed of sound and u_ℓ is the average local velocity in the region between the bow wave and the body. The second term is small compared to the first term for low supersonic speeds, so the expression can be simplified to:

$$t_r \simeq \frac{\Delta}{a_\infty(1-M_\ell)}$$

where a_∞ is the freestream speed of sound and M_ℓ is the average local Mach number between the bow wave and the body. This expression describes the flow response phenomena observed in the aeroballistic range tests with a decelerating body very near Mach 1. If it is assumed that the transient response to a Mach number oscillation in the freestream is similar, and that an oscillation with a period as small as four times the response time, t_r, may be observed, an estimate of the frequency response of this system can be made. The calculation of the frequency response of the shock detachment distance is given in Fig. 3. As the Mach number increases, the frequency response becomes high, primarily because of the very short distance between the shock and the body and a resultant short response time. This is offset, however, by a reduction in the Mach number fluctuation that can be resolved, (see Fig. 1). The spectral density of the Mach number variation could thus be determined for correlation with the more conventional fluctuation measurements in the domain of validity. An independent measurement of the Mach number fluctuation in addition to the measurement of other fluctuating aerodynamic properties would be most useful even if the precision of standoff distance measurement limited the comparison to a very low supersonic Mach number (say $M_\infty \sim 1.05$ to 1.1) and the lower portion of the frequency spectrum which exists in modern transonic wind tunnels (frequency < 1000 Hz).

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Natural Frequencies of a Cantilever with Slender Tip Mass

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THE exact frequency equation for a uniform cantilever beam carrying a tip mass has been considered by Pipes,[1] Prescott,[2] Temple and Bickley,[3] and Durvasula.[4] In

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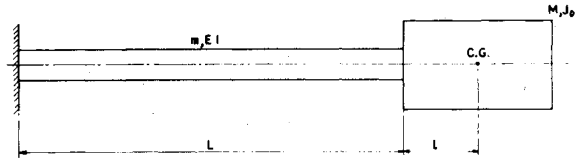

Fig. 1 Schematic of a cantilever with a tip mass.

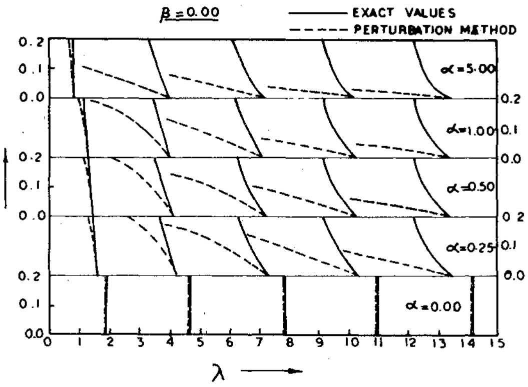

Fig. 2 Comparison of natural frequencies by perturbation method with exact values, $\beta = 0.00$.

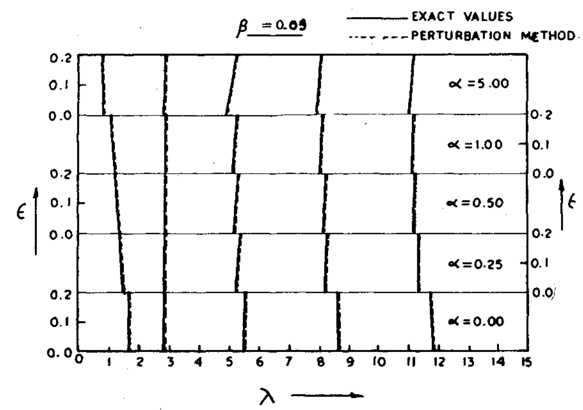

Fig. 3 Comparison of natural frequencies by perturbation method with exact values, $\beta = 0.05$.

some situations, the tip mass may be slender in the axial direction, and the center of gravity of the tip mass and its point of attachment to the beam may not coincide. These problems may arise, for instance, in wind-tunnel stings carrying an airplane or a missile model, in large aspect ratio wings carrying heavy tip tanks, in Stockbridge dampers used for damping out galloping of transmission lines, or in launch vehicles with payloads at the tip. In the present Note, the exact frequency equation for such a case is solved with a digital computer, and the results are compared with a perturbation solution given by Bhat and Wagner.[5]

A schematic sketch of the uniform beam, carrying slenver tip mass, is shown in Fig. 1. The equation of motion for small deflections is

$$EIy''''(\kappa,t) + m\ddot{y}(\kappa,t) = 0 \tag{1}$$

where primes and dots denote differentiation with regard to space and time, respectively. The boundary conditions at the tip are obtained by variational methods as

$$EIy''(L,t) = -(J_0+M\ell^2)\ddot{y}'(L,t) - M\ell\ddot{y}(L,t) \tag{2}$$

$$EIy'''(L,t) = M\ddot{y}(L,t)M\ell\ddot{y}'(L,t) \tag{3}$$

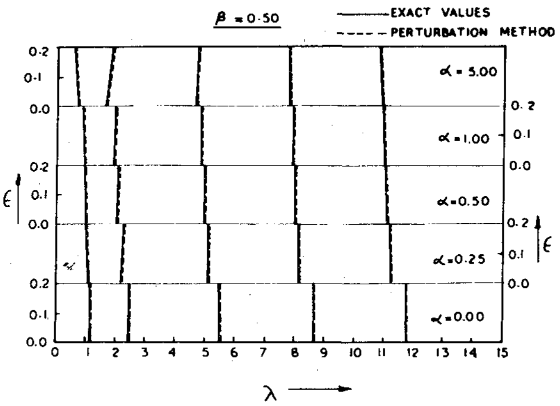

Fig. 4 Comparison of natural frequencies by perturbation method with exact values, $\beta=0.50$.

where L is the length of the beam, and ℓ is the distance between point of attachment and the center of gravity of the tip mass. For free harmonic vibrations with frequency ω, the exact frequency equation is obtained as

$$1+\alpha\beta\lambda^4+(1-\alpha\beta\lambda^4)G(\lambda)+\alpha\lambda[H(\lambda)-J(\lambda)] -\beta\lambda^3[H(\lambda)+J(\lambda)]-2\alpha\epsilon\lambda^2F(\lambda) -\alpha\epsilon^2\lambda^3[H(\lambda)+J(\lambda)]=0 \quad (4)$$

where

$$F(\lambda)=\sinh\lambda\cdot\sin\lambda,\quad G(\lambda)=\cosh\lambda\cdot\cos\lambda,$$
$$H(\lambda)=\sinh\lambda\cdot\cos\lambda,\quad J(\lambda)=\cosh\lambda\cdot\sin\lambda$$

and

$$\lambda^4=m\omega^2L^4/EI,\quad \alpha=M/mL,\quad \epsilon=\ell/L,\quad \beta=J_0/mL^3.$$

The solution of the frequency equation given by Eq. (4), for small values of ϵ, was obtained by a perturbation procedure,[5] where the frequencies can be obtained as

$$\lambda=\lambda_0+\epsilon\lambda_1+\epsilon^2\lambda_2+\ldots\ldots \quad (5)$$

where

$$\lambda_1=2\alpha\lambda_0^2F(\lambda_0)/\{4\beta\lambda_0^3[1-G(\lambda_0)]+\alpha(1-\beta\lambda_0^4)\times \times[H(\lambda_0)-J(\lambda_0)]-2\alpha\lambda_0F(\lambda_0)-2\beta\lambda_0^3G(\lambda_0) -3\beta\lambda_0^2[H(\lambda_0)+J(\lambda_0)]\} \quad (6)$$

and

$$\lambda_2=\{(4\alpha\lambda_0\lambda_1+2\alpha\lambda_1^2-\alpha\beta\lambda_0^4\lambda_1^2)F(\lambda_0) +(\beta\lambda_0^3\lambda_1^2+4\alpha\beta\lambda_0^3\lambda_1^2)[H(\lambda_0)-J(\lambda_0)] +6\beta\lambda_0^2\lambda_1^2G(\lambda_0)+(\lambda_0\lambda_1^2+3\beta\lambda_0\lambda_1^2 -2\alpha\lambda_0^2\lambda_1+\alpha\lambda_0^3)[H(\lambda_0)+J(\lambda_0)] -6\alpha\beta\lambda_0^2\lambda_1^2[1-G(\lambda_0)]\}/ \{4\alpha\beta\lambda_0^3[1-G(\lambda_0)]+\alpha(1-\beta\lambda_0^4)[H(\lambda_0)-J(\lambda_0)] -2\lambda_0F(\lambda_0)-3\beta\lambda_0^2[H(\lambda_0)+J(\lambda_0)] -2\beta\lambda_0^3G(\lambda_0)\} \quad (7)$$

The exact values of the natural frequencies for the first five modes, obtained by solving Eq. (4) in a digital computer, are compared with those obtained by the perturbation method, for various values of ϵ, in Figs. 2-4. It can be seen that, except in Fig. 2 for the case of $\beta=0.0$, the perturbation method gives good results. Every mass will have reasonable moment of inertia, i.e., β will not be zero in practical cases. Hence, the perturbation method can be used to get natural frequencies with good accuracy.

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Shock Detachment Distance at Near Sonic Speeds

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THIS Note documents some recent experimental observations of the shock detachment distance for spherical and spherically blunted bodies at near-sonic speeds ($1.0<M_\infty<1.5$) in air. Spheres and spherically blunted cones have been tested in the 10-ft-diam AEDC Aeroballistic Range G to determine their transonic drag characteristics. The bow shock ahead of the body is clearly visible in shadowgraph pictures taken of the model in flight which are used to establish the time-distance relationship and drag. A sequence of photographs illustrating the variation of shock detachment distance with Mach number for the sphere is shown in Fig. 1. This 1.5-in.-diam sphere was tested at a solid blockage of 0.016% which is considered to be interference free. The Reynolds number based on diameter, Re_d for this study was approximately 1×10^6. A sequence of photographs of the shock detachment variation with Mach number for a 10° semiangle, 0.3 bluntness ratio, d_n/d, cone is displayed in Fig. 2. This 3-in. base-diam cone was tested at a solid blockage based on the model base diameter d of 0.0625%. The Re_d was approximately 2×10^6 for these tests. The results presented herein are free of the influence of (1) the blast wave which emanates from the muzzle of the model launcher, and (2) deceleration of the model during flight. Both of these problems can be encountered in testing in aeroballistic ranges at speeds very near Mach 1 and involving rapid decelerations. The presence of a blast wave just ahead of a model results in an induced velocity field and a reduced effective Mach number. Rapid deceleration of the model results in a mislocation of the shock for any instantaneous Mach number, since a finite flow response time is invloved. Such effects were avoided in the present study by careful selection of model mass and launch velocity.

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Index categories: Subsonic and Transonic Flow; Supersonic and Hypersonic Flow.

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